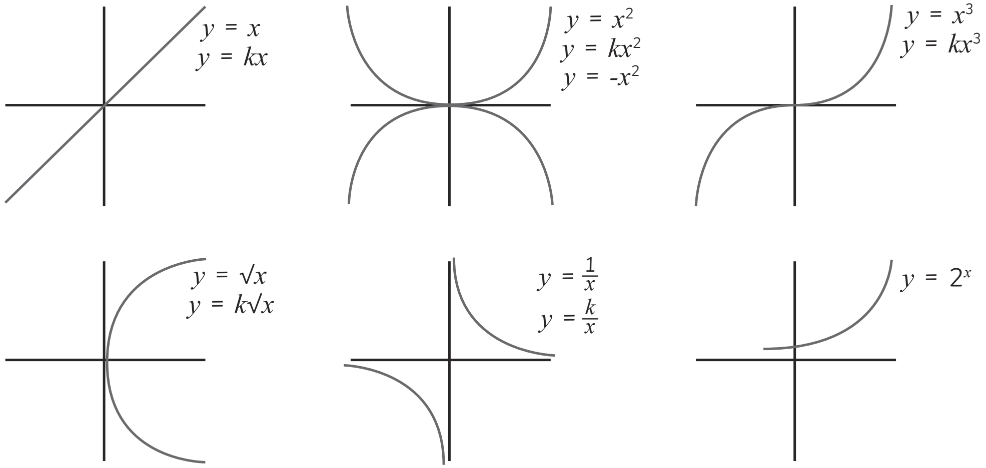
Graphs

Make sure that you can recognise these graphs.



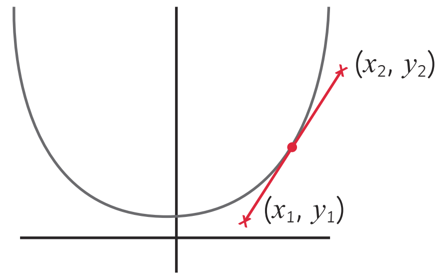
The equation of a **circle** with centre (0, 0) and radius r is given by $x^2 + y^2 = r^2$

Don't forget that the tangent to a circle will always be perpendicular to its radius.

Gradient

To find the gradient of a curve at a point, we draw the tangent at that point and find the gradient of the tangent.

Remember, gradient $= \frac{y_2 - y_1}{x_2 - x_1}$



Functions

A **composite function** is created by finding the function of a function. For $fg(x)$ we apply the function $g(x)$ first, then apply $f(x)$ to the answer.

e.g. $f(x) = 3x$ and $g(x) = 2x - 5$

$fg(x) = 3(2x - 5) = 6x - 15$

An **inverse function** is the reverse of a function. Swap the x and y and rearrange to make y the subject.

Solving Equations

To solve a quadratic equation **without** a calculator you can factorise and then solve.

E.g. Solve $3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

$3x + 2 = 0$ or $x - 1 = 0$

$x = -\frac{2}{3}$ or $x = 1$

Note: on a graph, these values show where it crosses the x-axis.

Quadratic Sequences

A quadratic sequence is one which has a common second difference. We halve the second difference to find the coefficient of x^2.

Geometric Sequences

A geometric sequence is one in which each term is found by multiplying the term before it by a common ratio, r.

Quadratic Simultaneous Equations

Make x or y the subject of the linear equation and **substitute** it into the quadratic equation. Don't forget to calculate the value of both letters!

Learn all the foundation key facts and remember these top tips!

Iteration

This is a way of finding approximate solutions to equations without using trial and improvement. Make sure you use your calculator to help you!

An iteration formula might look like this:

$x_{n+1} = 1 + \frac{11}{x_n - 3}$

You will be given a starting point, e.g. $x_1 = -2$

We can use this starting point to find an estimate for the solution.

$x_2 = 1 + \frac{11}{-2 - 3}$

$x_2 = -1.2$

$x_3 = 1 + \frac{11}{-1.2 - 3}$

$x_3 = -1.61...$

$x_4 = 1 + \frac{11}{-1.61... - 3}$

$x_4 = -1.38...$

Keep going until you have the required level of accuracy.

To 2 decimal places $x = 1.46$

Straight Line Graphs

Two lines are perpendicular if their gradients have a product of -1.

E.g. 4 and $-\frac{1}{4}$

$-\frac{3}{2}$ and $\frac{2}{3}$

Expanding Three Brackets

To expand three brackets, start by expanding two and then multiplying each term by both parts of the third bracket.

E.g. $(x + 1)(x + 2)(x + 3) = (x^2 + 3x + 2)(x + 3)$

$= x^3 + 3x^2 + 3x^2 + 9x + 2x + 6$

$= x^3 + 6x^2 + 11x + 6$

Algebraic Fractions

To simplify an algebraic fraction, factorise both the numerator and the denominator and 'cancel' the common factors.

E.g. $\frac{3x + 6}{2x^2 + 3x - 2} = \frac{3(x + 2)}{(2x - 1)(x + 2)} = \frac{3}{2x - 1}$

Quadratic Equations

The solutions of $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

We can complete the square on the expression $x^2 + bx + c$ by first halving the coefficient of x, then squaring it and subtracting.

E.g. $x^2 + 6x - 2 = (x + 3)^2 - 2 - 9$

$= (x + 3)^2 - 11$

The turning point of this graph is

(-3, -11).

We can also solve $x^2 + 6x - 2 = 0$ using its completed square form.

$(x + 3)^2 - 11 = 0$

$(x + 3)^2 = 11$

$x + 3 = \pm\sqrt{11}$

$x = \sqrt{11} - 3$ or $x = -\sqrt{11} - 3$

Proof

An **even** number is given by $2n$

An **odd** number is given by $2n + 1$

Consecutive means one after the other

Sum means add

Product means multiply