

Answers

Calculator

GCSE Maths

Revision

Higher

Booklet

2

BEYOND
MATHS



Calculator

GCSE Maths Revision

Higher Booklet 2 **Answers**

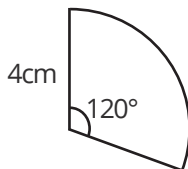
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1. Simplify $\frac{x+1}{4x+4}$

$$\frac{1}{4}$$

2. The diagram shows a sector of a circle of radius 4cm. Calculate the area of the sector, giving your answer correct to 3 significant figures.



$$\frac{120}{360} \times \pi \times 4^2 = 16.8\text{cm}^2 \text{ (to 3 significant figures)}$$

3. Bella invests £800 in a bank account which offers 2% compound interest per annum. How much will Bella have in her bank account after 3 years?

$$800 \times 1.02^3 = \text{£}848.97 \text{ (to the nearest penny)}$$

4. Factorise fully: $x^2 + 10x + 21$

$$(x + 3)(x + 7)$$

5. Write $0.\dot{7}$ as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 0.77777\dots$$

$$10x = 7.77777\dots$$

$$9x = 7$$

$$x = \frac{7}{9}$$

6. Solve $4x + 1 < 10$

$$4x < 9$$

$$x < \frac{9}{4} \text{ or } 2.25$$

7. Prove that $(n + 1)^2 - (n - 1)^2 = 4n$

$$(n + 1)^2 = n^2 + 2n + 1$$

$$(n - 1)^2 = n^2 - 2n + 1$$

$$n^2 + 2n + 1 - (n^2 - 2n + 1) = 4n$$

8. Make x the subject: $5x + y = pq$

$$5x = pq - y$$

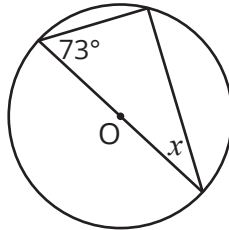
$$x = \frac{pq - y}{5}$$

9. Find the equation of the line that passes through the points with coordinates $(0, 1)$ and $(2, 5)$.

$$m = \frac{5 - 1}{2 - 0} = 2$$

$$y = 2x + 1$$

10. The diagram shows a circle centred at O . Work out the size of the angle marked x , giving reasons for each stage of your working.

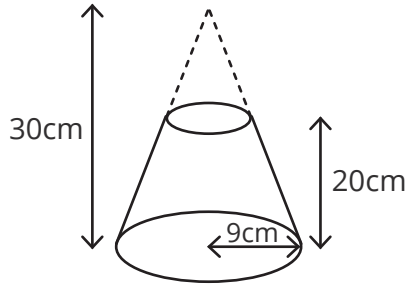


The angle in a semi-circle is 90° (or similar). Angles in a triangle add to 180° .

$$x = 180 - 90 - 73$$

$$x = 17^\circ$$

11. The diagram shows the frustum of a cone. The height of the cone is 30cm and the height of the frustum is 20cm. The radius of the base of the cone is 9cm. Work out the volume of the frustum, giving your answer in terms of π .



$$\text{Scale factor} = 30 \div 10 = 3$$

$$\text{Radius of 'top' cone} = 9 \div 3 = 3\text{cm}$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume} = \frac{1}{3} \times \pi \times 9^2 \times 30 - \frac{1}{3} \times \pi \times 3^2 \times 10 = 780\pi\text{cm}^3$$

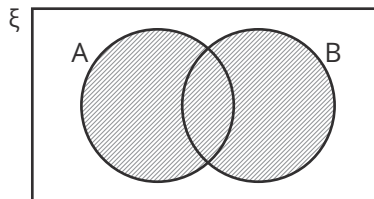
12. $a:b = 3:2$ and $b:c = 4:7$. Write the ratio $a:c$ in its simplest form.

$$a:b = 6:4$$

$$a:b:c = 6:4:7$$

$$a:c = 6:7$$

13. On the Venn diagram, shade $A \cup B$.

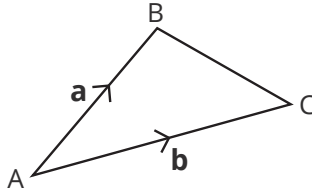


14. Solve $2x^2 + 3x - 4 = 0$, giving your answer correct to 1 decimal place.

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-4)}}{2 \times 2}$$

$$x = 0.9 \text{ or } -2.4$$

15. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$. Write down the vector \overrightarrow{BC} .



$$\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$$

16. Solve the simultaneous equations.

$$x + 3y = 5$$

$$x + y = 1$$

$$2y = 4$$

$$y = 2$$

$$x + 2 = 1$$

$$x = -1$$

17. Write as a single power of 2:

$$4^3$$

$$(2^2)^3 = 2^6$$

18. Solve the equation $\sin(x) = 0.5$ for $0^\circ \leq x \leq 360^\circ$.

$$x = \sin^{-1}(0.5) = 30^\circ$$

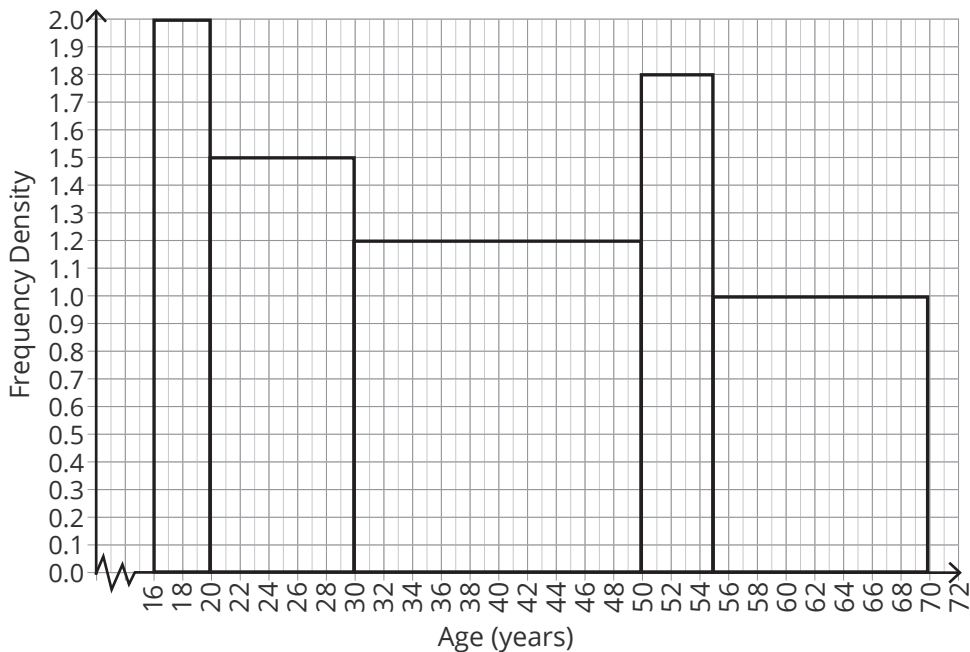
$$x = 30^\circ, 150^\circ$$

19. Work out the value of $(3 \times 10^4) \times (7 \times 10^5)$, giving your answer in standard form.

$$21 \times 10^9 = 2.1 \times 10^{10}$$

20. The histogram shows information about the ages of some employees in a company. Draw a histogram representing this information.

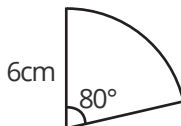
| Age (x years) | Frequency | Frequency Density |
|------------------|-----------|--------------------------------------|
| $16 \leq x < 20$ | 8 | $8 \div 4 = 2$ |
| $20 \leq x < 30$ | 15 | $15 \div 10 = 1.5$ |
| $30 \leq x < 50$ | 24 | $24 \div 20 = 1.2$ |
| $50 \leq x < 55$ | 9 | $9 \div 5 = 1.8$ |
| $55 \leq x < 70$ | 15 | $15 \div 15 = 1$ |



1. Simplify $\frac{x+5}{3x^2+15x}$

$$\frac{1}{3x}$$

2. The diagram shows a sector of a circle of radius 6cm. Calculate the area of the sector, giving your answer correct to 3 significant figures.



$$\frac{80}{360} \times \pi \times 6^2 = 25.1\text{cm}^2 \text{ (to 3 significant figures)}$$

3. Bella invests £1200 in a bank account which offers 3.5% compound interest per annum. How much will Bella have in her bank account after 4 years?

$$1200 \times 1.035^4 = \text{£}1377.03 \text{ (to the nearest penny)}$$

4. Factorise fully: $x^2 - 6x - 16$

$$(x + 2)(x - 8)$$

5. Write $0.\dot{1}\dot{3}$ as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 0.131313\dots$$

$$100x = 13.131313\dots$$

$$99x = 13$$

$$x = \frac{13}{99}$$

6. Solve $-4 \leq 5x + 1 < 27$

$$-5 \leq 5x < 26$$

$$-1 \leq x < 5.2$$

7. Prove that $(n + 2)^2 - (n - 2)^2$ is a multiple of 4 for all integer values of n .

$$(n + 2)^2 = n^2 + 4n + 4$$

$$(n - 2)^2 = n^2 - 4n + 4$$

$$n^2 + 4n + 4 - (n^2 - 4n + 4) = 8n$$

$8n = 4(2n)$ which is, by definition, a multiple of 4.

8. Make x the subject: $\frac{2x}{y} = q$

$$2x = qy$$

$$x = \frac{qy}{2}$$

9. Find the equation of the line that passes through the points with coordinates (2, 3) and (10, 5).

$$m = \frac{5 - 3}{10 - 2} = \frac{1}{4}$$

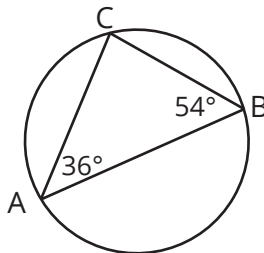
$$y = \frac{1}{4}x + c$$

$$3 = \frac{1}{4} \times 2 + c$$

$$c = \frac{5}{2}$$

$$y = \frac{1}{4}x + \frac{5}{2}$$

10. The diagram shows a triangle inscribed in a circle. Is the line AB the diameter of the circle? Give reasons for your answer.

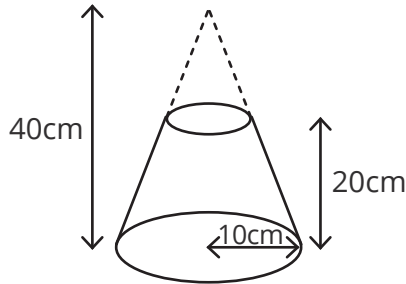


The angle in a semi-circle is 90° (or similar), and angles in a triangle add to 180° .

$180 - 36 - 54 = 90^\circ$, as required.

Yes, AB is the diameter of the circle.

11. The diagram shows the frustum of a cone. The height of the cone is 40cm and the height of the frustum is 20cm. The radius of the base of the cone is 10cm. Work out the volume of the frustum, giving your answer in terms of π .



$$\text{Scale factor} = 40 \div 20 = 2$$

$$\text{Radius of 'top' cone} = 10 \div 2 = 5\text{cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume} = \frac{1}{3} \times \pi \times 10^2 \times 40 - \frac{1}{3} \times \pi \times 5^2 \times 20 = \frac{3500}{3} \pi \text{cm}^3$$

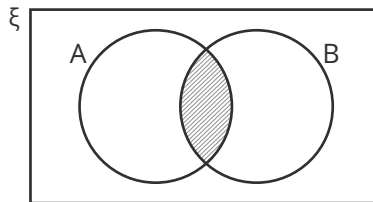
12. $a:b = 4:3$ and $b:c = 5:2$. Write the ratio $a:c$ in its simplest form.

$$a:b = 20:15$$

$$b:c = 15:6$$

$$a:c = 20:6 = 10:3$$

13. On the Venn diagram, shade $A \cap B$.



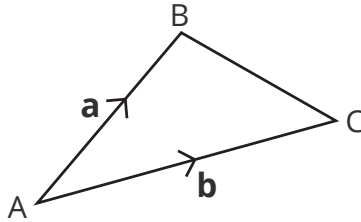
14. Solve $3x^2 - x - 5 = 0$, giving your answer correct to 1 decimal place.

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$x = -1.1 \text{ or } 1.5$$

15. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$. Point P lies at the midpoint of BC.

Write down the vector \overrightarrow{AP} .



$$\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$$

$$\overrightarrow{AP} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{b} + \mathbf{a})$$

16. Solve the simultaneous equations.

$$2x + 3y = 0$$

$$x - y = 5$$

$$5x = 15$$

$$x = 3$$

$$3 - y = 5$$

$$y = -2$$

17. Write as a single power of 2: $\frac{1}{8}$

$$(2^3)^{-1} = 2^{-3}$$

18. Solve the equation $\cos(x) = 1$ for $0^\circ \leq x \leq 360^\circ$.

$$x = \cos^{-1}(1) = 0^\circ$$

$$x = 0^\circ, 360^\circ$$

19. Work out the value of $(4 \times 10^6) \div (8 \times 10^2)$, giving your answer in standard form.

$$0.5 \times 10^4 = 5 \times 10^3$$

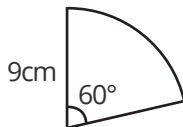
20. The table shows information about the ages of some employees. Fill in the missing gaps.

| Age (x years) | Frequency | Frequency Density |
|------------------|--|--------------------------------------|
| $16 \leq x < 20$ | 9 | $9 \div 4 = 2.25$ |
| $20 \leq x < 30$ | $1.5 \times 10 = 15$ | 1.5 |
| $30 \leq x < 50$ | 7 | $7 \div 20 = 0.35$ |
| $50 \leq x < 55$ | $2.2 \times 5 = 11$ | 2.2 |
| $55 \leq x < 70$ | 9 | $9 \div 15 = 0.6$ |

1. Simplify $\frac{x+6}{x^2+10x+24}$

$$\frac{1}{x+4}$$

2. The diagram shows a sector of a circle of radius 9cm. Calculate the arc length of the sector, giving your answer correct to 3 significant figures.



$$\frac{60}{360} \times \pi \times 18 = 9.42\text{cm (to 3 significant figures)}$$

3. Bella invests £500 in a bank account which offers 0.12% compound interest per annum. How much will Bella have in her bank account after 3 years?

$$500 \times 1.0012^3 = \text{£}501.80 \text{ (to the nearest penny)}$$

4. Factorise fully: $x^2 - 36$

$$(x - 6)(x + 6)$$

5. Write 0.25 as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 0.25555\dots$$

$$100x = 25.555\dots$$

$$10x = 2.555\dots$$

$$90x = 23$$

$$x = \frac{23}{90}$$

6. Solve $8 - 2x > 3$

$$-2x > -5$$

$$x < 2.5$$

7. Prove that $n^2 + 2n + 1$ is always a square number for integer values of n .

$$n^2 + 2n + 1 = (n + 1)(n + 1) = (n + 1)^2$$

This is a square number.

8. Make x the subject: $\frac{w+x}{u} = y$

$$w + x = uy$$

$$x = uy - w$$

9. Find the equation of the line that passes through the points with coordinates (4, -1) and (2, 3).

$$m = \frac{3 - (-1)}{2 - 4} = \frac{4}{-2}$$

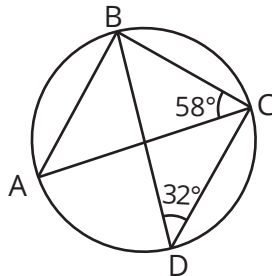
$$y = -2x + c$$

$$3 = -2 \times 2 + c$$

$$c = 7$$

$$y = -2x + 7$$

10. The diagram shows triangles ABC and BCD inscribed inside a circle. Is AC the diameter of the circle? You must give reasons for your answer.

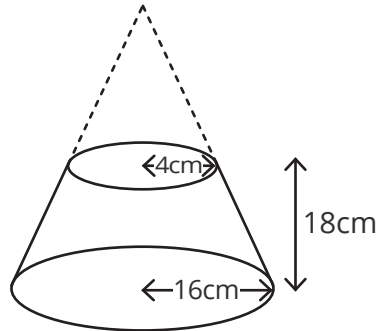


Angle BAC = 32° since angles in the same segment are equal.

Angle ABC = 180 - 58 - 32 = 90° since angles in a triangle sum to 180°.

We know that the angle subtended from the diameter is 90° therefore AC must be the diameter of the circle.

11. A frustum is made by removing a small cone from a larger cone as shown. Work out the volume of the frustum, giving your answer correct to 3 significant figures.



$$\text{Scale factor} = 16 \div 4 = 4$$

$$\text{Height of small cone} = x$$

$$\text{Height of large cone} = 18 + x$$

$$18 + x = 4x$$

$$x = 6$$

$$\text{Volume of small cone} = \frac{1}{3} \times \pi \times 4^2 \times 6 = 32\pi\text{cm}^3$$

$$\text{Volume of large cone} = \frac{1}{3} \times \pi \times 16^2 \times 24 = 2048\pi\text{cm}^3$$

$$\text{Volume of frustum} = 2048\pi - 32\pi = 6333.45\dots\text{cm}^3$$

Volume is 6330cm^3 correct to 3 significant figures.

12. $a:b = 4:3$ and $b:c = 5:2$. Write a as a fraction of c , giving your answer in the form $a = \frac{x}{y}c$, where x and y are integers.

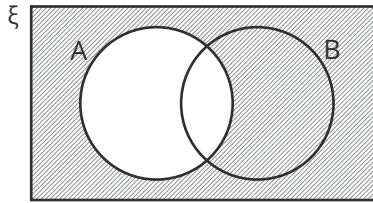
$$a:b = 20:15$$

$$b:c = 15:6$$

$$a:c = 20:6 = 10:3$$

$$a = \frac{10}{3}c$$

13. On the Venn diagram, shade A' .



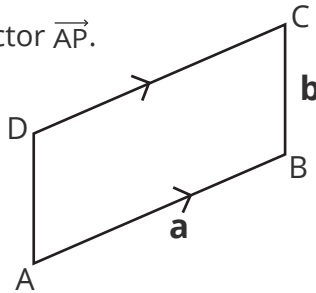
14. Solve $2x^2 + x = 4$, giving your answer correct to 1 decimal place.

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times (-4)}}{2 \times 2}$$

$$x = 1.2 \text{ or } -1.7$$

15. ABCD is a parallelogram. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. Point P lies at the midpoint of DC.

Write down the vector \overrightarrow{AP} .



$$\overrightarrow{AP} = \mathbf{b} + \frac{1}{2}\mathbf{a}$$

16. Solve the simultaneous equations.

$$3x + 4y = 19$$

$$4x + 3y = 23$$

$$16x + 12y = 92$$

$$-9x + 12y = 57$$

$$\hline 7x = 35$$

$$x = 5$$

$$15 + 4y = 19$$

$$4y = 4$$

$$y = 1$$

17. Write as a single power of 2:

$$4^2 \times \left(\frac{1}{2}\right)^{-1}$$

$$(2^2)^2 \times 2^1 = 2^5$$

18. Solve the equation $\tan(x) = 1$ for $0^\circ \leq x \leq 360^\circ$.

$$x = \tan^{-1}(1) = 45^\circ$$

$$x = 45^\circ, 225^\circ$$

19. Work out the value of $(3 \times 10^4)^3$, giving your answer in standard form.

$$27 \times 10^{12} = 2.7 \times 10^{13}$$

20. The table shows information about the ages of some employees in a company. Fill in the missing gaps and explain how you know that a mistake has been made.

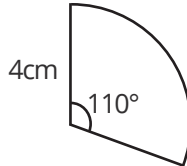
| Age (x years) | Frequency | Frequency Density |
|------------------|----------------------|--------------------|
| $16 \leq x < 24$ | 10 | $10 \div 8 = 1.25$ |
| $24 \leq x < 30$ | $0.8 \times 6 = 4.8$ | 0.8 |
| $30 \leq x < 50$ | 18 | $18 \div 20 = 0.9$ |
| $50 \leq x < 60$ | $1.4 \times 10 = 14$ | 1.4 |
| $60 \leq x < 75$ | 12 | $12 \div 15 = 0.8$ |

You cannot have a frequency of 4.8.

1. Simplify $\frac{2x-6}{x^2+x-12}$

$$\frac{2}{x+4}$$

2. The diagram shows a sector of a circle of radius 4cm. Calculate the arc length of the sector, giving your answer correct to 3 significant figures.



$$\frac{110}{360} \times \pi \times 8 = 7.68\text{cm (to 3 significant figures)}$$

3. Bella invests £500 in a bank account which offers 2.1% compound interest per annum. After how many years will Bella have more than £600?

$$500 \times 1.021^8 = \text{£}590.44 \text{ (to the nearest penny)}$$

$$500 \times 1.021^9 = \text{£}602.84 \text{ (to the nearest penny)}$$

9 years.

4. Factorise fully: $4x^2 - 25$

$$(2x - 5)(2x + 5)$$

5. Write $0.2\dot{5}1$ as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 0.251515151\dots$$

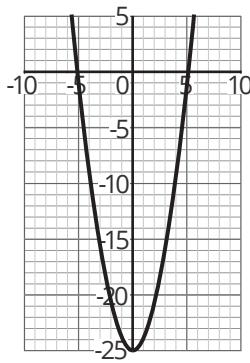
$$1000x = 251.515151\dots$$

$$10x = 2.515151\dots$$

$$990x = 249$$

$$x = \frac{249}{990} = \frac{83}{330}$$

6. Solve $3x^2 \geq 75$



$$x^2 \geq 25$$

$$x^2 - 25 \geq 0$$

$$(x - 5)(x + 5) \geq 0$$

$$x \leq -5 \text{ and } x \geq 5$$

7a. Write $x^2 - 2x + 7$ in the form $(x + a)^2 + b$, where a and b are integers.

$$x^2 - 2x + 7 = (x - 1)^2 - 1 + 7 = (x - 1)^2 + 6$$

7b. Hence, prove that $x^2 - 2x + 7$ is positive for all real values of x .

$(x - 1)^2$ is positive (since any real number squared is always positive). A positive plus a positive is positive, so $x^2 - 2x + 7$ is positive.

8. Make x the subject: $(x + y)^2 = t$

$$x + y = \sqrt{t}$$

$$x = \sqrt{t} - y$$

9. A line segment starts at the point $(0, 0)$ and finishes at $(2, 4)$. Find the equation of the perpendicular bisector to this line segment.

$$m_1 = \frac{4 - 0}{2 - 0} = 2$$

$$m_2 = -\frac{1}{2}$$

Midpoint is $(1, 2)$.

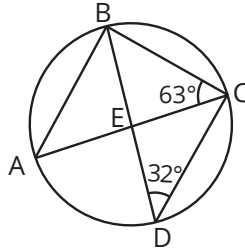
$$y = -\frac{1}{2}x + c$$

$$2 = -\frac{1}{2} \times 1 + c$$

$$c = \frac{5}{2}$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

10. The diagram shows triangles ABC and BCD inscribed inside a circle, such that BD is the diameter of the circle. Find the size of angle CED, giving reasons at each stage of your working.

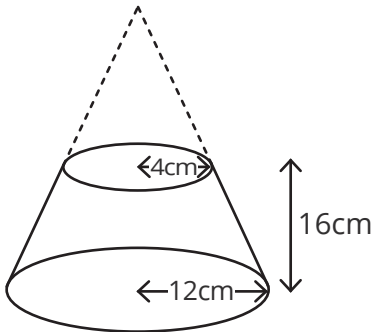


Angle BCD = 90° since angles in a semi-circle are 90°.

Angle ECD = 90 – 63 = 27°.

Angle CED = 180 – (32 + 27) = 121° since angles in a triangle sum to 180°.

11. A frustum is made by removing a small cone from a larger cone as shown. Work out the volume of the frustum, giving your answer correct to 3 significant figures.



Scale factor = 12 ÷ 4 = 3

Height of small cone = x

Height of large cone = 16 + x

16 + x = 3 x

x = 8

Volume of small cone = $\frac{1}{3} \times \pi \times 4^2 \times 8 = \frac{128}{3}\pi\text{cm}^3$

Volume of large cone = $\frac{1}{3} \times \pi \times 12^2 \times 24 = 1152\pi\text{cm}^3$

Volume of frustum = $1152\pi - \frac{128}{3}\pi = 3485.07\dots \text{cm}^3$

Volume is 3490cm³ correct to 3 significant figures.

12. $a:b = 5:7$ and $b:c = 2:3$. Write a as a fraction of c , giving your answer in the form $a = \frac{x}{y}c$, where x and y are integers.

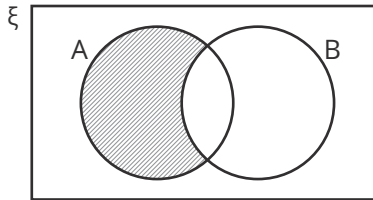
$$a:b = 10:14$$

$$b:c = 14:21$$

$$a:c = 10:21$$

$$a = \frac{10}{21}c$$

13. On the Venn diagram, shade $A \cap B'$.

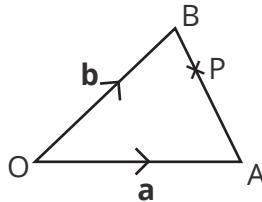


14. Solve $x^2 = 5x + 9$, giving your answer correct to 1 decimal place.

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

$$x = 6.4 \text{ or } -1.4$$

15. $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. Point P lies on AB such that $AP:PB = 4:1$. Write down the vector \vec{AP} .



$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

$$\vec{AP} = \frac{4}{5}(\mathbf{b} - \mathbf{a})$$

16. Solve the simultaneous equations.

$$6x - 4y = 22$$

$$4x + 5y = 7$$

$$\begin{array}{r} 12x - 8y = 44 \\ - 12x + 15y = 21 \\ \hline -23y = 23 \end{array}$$

$$y = -1$$

$$4x - 5 = 7$$

$$4x = 12$$

$$x = 3$$

17. Write as a single power of 2: $\frac{1}{4} \times 16^4$

$$2^{-2} \times (2^4)^4 = 2^{-2} \times 2^{16} = 2^{14}$$

18. Solve the equation $\cos(x) = 0.2$ for $0^\circ \leq x \leq 360^\circ$.

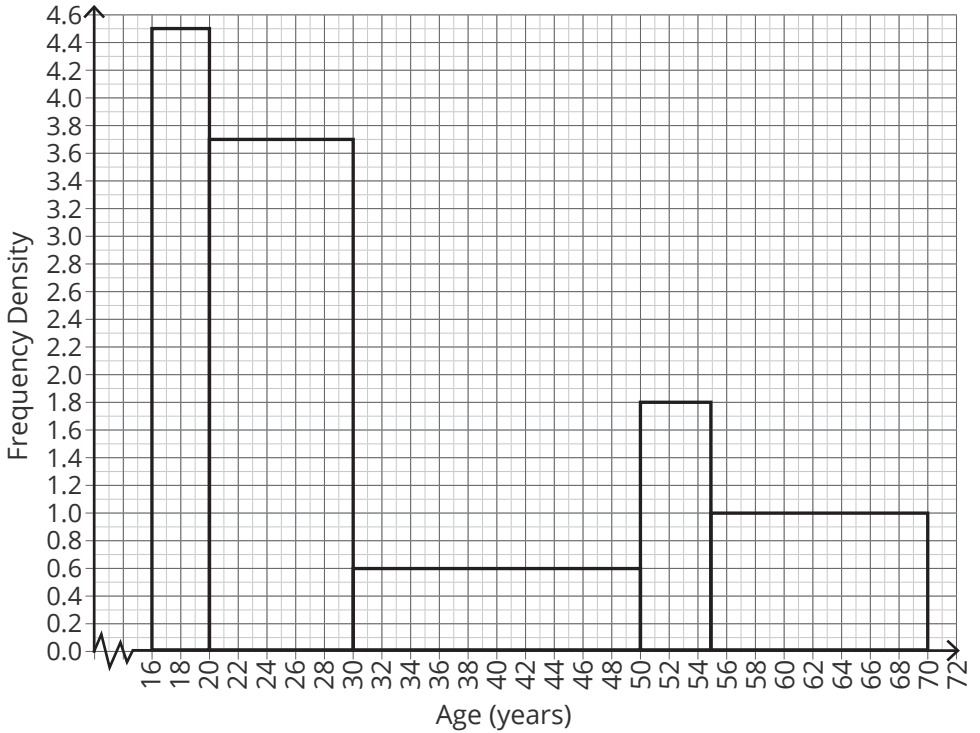
$$x = \cos^{-1}(0.2) = 78.5^\circ$$

$$x = 78.5^\circ, 281.5^\circ \text{ (correct to 1d.p.)}$$

19. Work out the value of $2.1 \times 10^4 + 3 \times 10^2$, giving your answer in standard form.

$$21\,300 = 2.13 \times 10^4$$

20. The histogram shows information about the ages of some employees in a company. Calculate the frequencies for each group.

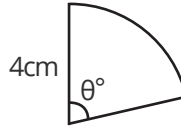


| Age (x years) | Frequency | Frequency Density |
|------------------|----------------------|-------------------|
| $16 \leq x < 20$ | $4.5 \times 4 = 18$ | 4.5 |
| $20 \leq x < 30$ | $3.7 \times 10 = 37$ | 3.7 |
| $30 \leq x < 50$ | $0.6 \times 20 = 12$ | 0.6 |
| $50 \leq x < 55$ | $1.8 \times 5 = 9$ | 1.8 |
| $55 \leq x < 70$ | $1 \times 15 = 15$ | 1 |

1. Simplify $\frac{x^2 - x - 20}{x^2 - 4x - 5}$

$$\frac{x + 4}{x + 1}$$

2. The diagram shows a sector of a circle of radius 4cm. Given that the area of the sector is 4.19cm^2 , work out the size of angle θ , giving your answer correct to 1 decimal place.



$$\frac{\theta}{360} \times \pi \times 4^2 = 4.19 \quad \theta = 30.0^\circ$$

3. Bella invests some money in a bank account which offers 2% compound interest per annum. She leaves her money in the account for 4 years. What is the single percentage increase in the money in her account?

$$1.02^4 = 1.0824\dots$$

This is an 8.2% increase correct to 1 decimal place.

4. Factorise fully: $49x^3 - 64x$

$$x(49x^2 - 64) = x(7x - 8)(7x + 8)$$

5. Write $0.\dot{1}3\dot{4}$ as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 0.1343434\dots$$

$$1000x = 134.343434\dots$$

$$10x = 1.343434\dots$$

$$990x = 133$$

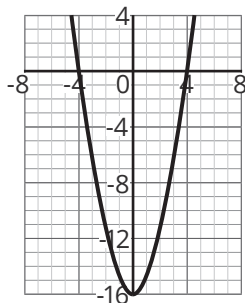
$$x = \frac{133}{990}$$

6. Solve $x^2 + 9 \leq 25$

$$x^2 - 16 \leq 0$$

$$(x - 4)(x + 4) \leq 0$$

$$x \leq -4 \text{ and } x \geq 4$$



7a. Write $x^2 - 4x + 6$ in the form $(x + a)^2 + b$, where a and b are integers.

$$x^2 - 4x + 6 = (x - 2)^2 - 4 + 6 = (x - 2)^2 + 2$$

7b. Hence, prove that $x^2 - 4x + 6$ is positive for real values of x .

$(x - 2)^2$ is positive (since any real number squared is always positive). A positive plus a positive is positive, so $x^2 - 4x + 6$ is positive.

8. Make x the subject: $(3x + y)^3 = t + p$

$$3x + y = \sqrt[3]{t + p}$$

$$x = \frac{\sqrt[3]{t + p} - y}{3}$$

9. A line segment starts at the point $(1, 3)$ and finishes at $(3, 5)$. Find the equation of the perpendicular bisector to this line segment.

$$m_1 = \frac{5 - 3}{3 - 1} = 1$$

$$m_2 = -1$$

Midpoint is $(2, 4)$.

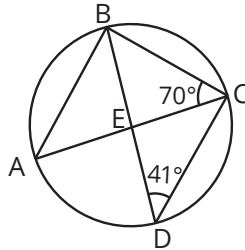
$$y = -x + c$$

$$4 = -2 + c$$

$$c = 6$$

$$y = -x + 6$$

10. The diagram shows triangles ABC and BCD inscribed inside a circle, such that BD is the diameter of the circle. Find angle CED, giving reasons at each stage of your working.

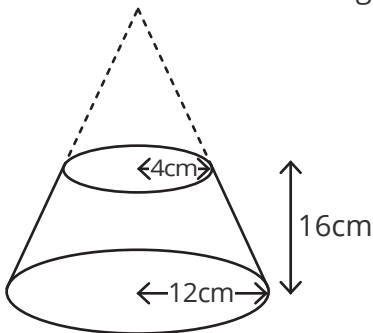


Angle BCD = 90° since angles in a semi-circle are 90° (or similar).

Angle ECD = 90 - 70 = 20°.

Angle CED = 180 - (20 + 41) = 119° since angles in a triangle sum to 180°.

11. A frustum is made by removing a small cone from a larger cone as shown. Calculate the curved surface area of the frustum. Give your answer correct to three significant figures.



$$\text{Scale factor} = 12 \div 4 = 3$$

$$\text{Height of small cone} = x$$

$$\text{Height of large cone} = 16 + x$$

$$16 + x = 3x$$

$$x = 8$$

$$\text{Sloped length of small cone} = \sqrt{8^2 + 4^2} = 4\sqrt{5}\text{cm}$$

$$\text{Sloped length of large cone} = \sqrt{24^2 + 12^2} = 12\sqrt{5}\text{cm}$$

$$\text{Curved surface area of small cone} = \pi \times 4 \times 4\sqrt{5} = 112.3\dots\text{cm}^2$$

$$\text{Curved surface area of large cone} = \pi \times 12 \times 12\sqrt{5} = 1011.5\dots\text{cm}^2$$

$$\text{Curved surface area of frustum} = 1011.5\dots - 112.3\dots = 899.1\dots\text{cm}^2$$

Curved surface area is 899cm² correct to 3 significant figures.

12. $a:b = 3:5$ and $b:c = 4:9$. Write a as a fraction of c , giving your answer in the form $a = \frac{x}{y}c$, where x and y are integers.

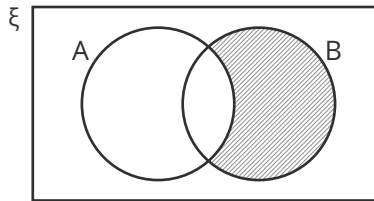
$$a:b = 12:20$$

$$b:c = 20:45$$

$$a:c = 12:45$$

$$a = \frac{12}{45}c \quad a = \frac{4}{15}c$$

13. On the Venn diagram, shade $A' \cap B$.



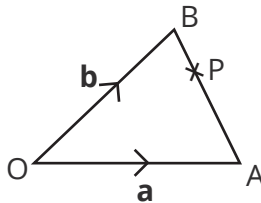
14. Solve $2x^2 = 2 - x$, giving your answer correct to 1 decimal place.

$$2x^2 + x - 2$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times (-2)}}{2 \times 2}$$

$$x = 0.8 \text{ or } -1.3$$

15. $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. Point P lies on AB such that $AP:PB = 3:2$. Write down the vector \vec{AP} .



$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

$$\vec{AP} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

16. Solve the simultaneous equations.

$$5x + 3y = -5$$

$$y = x + 1$$

$$5x + 3(x + 1) = -5$$

$$8x = -8$$

$$x = -1$$

$$y = -1 + 1 \quad y = 0$$

17. Write as a single power of 2:

$$\frac{1}{8} \times \left(\frac{1}{32}\right)^3$$

$$2^{-3} \times (2^{-5})^3 = 2^{-3} \times 2^{-15} = 2^{-18}$$

18. Solve the equation $\sin(x) = 0.15$ for $0^\circ \leq x \leq 360^\circ$.

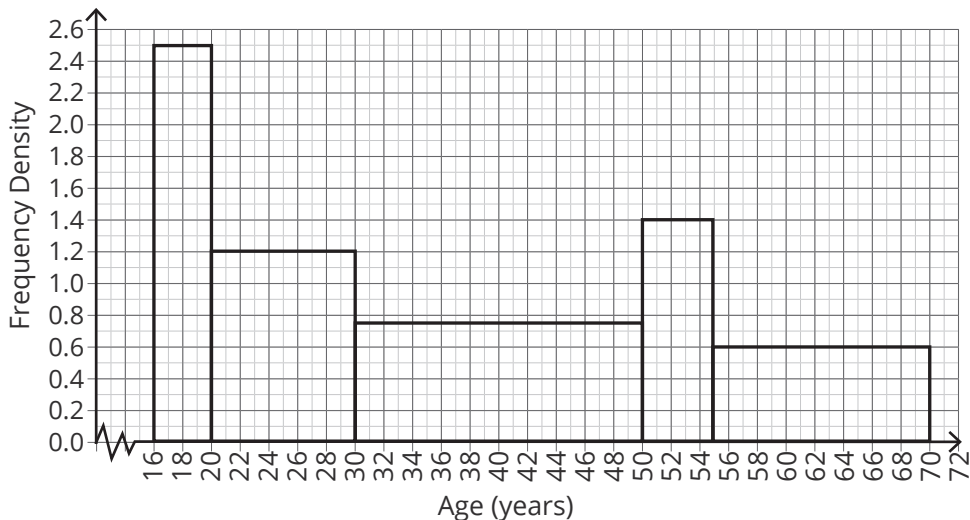
$$x = \sin^{-1}(0.15) = 8.6^\circ$$

$$x = 8.6^\circ, 171.4^\circ \text{ (correct to 1d.p.)}$$

19. Work out the value of $8.5 \times 10^4 - 3 \times 10^2$, giving your answer in standard form.

$$84\,700 = 8.47 \times 10^4$$

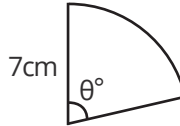
20. The histogram shows information about the ages of some employees in a company. Calculate the frequencies for each group.



| Age (x years) | Frequency | Frequency Density |
|------------------|---|-------------------|
| $16 \leq x < 20$ | $2.5 \times 4 = 10$ | 2.5 |
| $20 \leq x < 30$ | $1.2 \times 10 = 12$ | 1.2 |
| $30 \leq x < 50$ | $0.75 \times 20 = 15$ | 0.75 |
| $50 \leq x < 55$ | $1.4 \times 5 = 7$ | 1.4 |
| $55 \leq x < 70$ | $0.6 \times 15 = 9$ | 0.6 |

1. Simplify $\frac{x^2 - 49}{2x^2 + 17x + 21}$
 $\frac{x - 7}{2x + 3}$

2. The diagram shows a sector of a circle of radius 7cm. Given that the area of the sector is 10.69cm^2 , work out the size of angle θ , giving your answer correct to 1 decimal place.



$$\frac{\theta}{360} \times \pi \times 7^2 = 10.69 \quad \theta = 25.0^\circ$$

3. Bella invests some money in a bank account which offers 3% compound interest per annum. She leaves her money in the account for 3 years. What is the single percentage increase in the money in her account?

$$1.03^3 = 1.0927\dots$$

This is a 9.3% correct to 1 decimal place.

4. Factorise fully: $16x^2 - 49$

$$(4x - 7)(4x + 7)$$

5. Write $3.\dot{1}\dot{6}\dot{5}$ as a fraction in its simplest form. You must show your working.

$$\text{Let } x = 3.1656565\dots$$

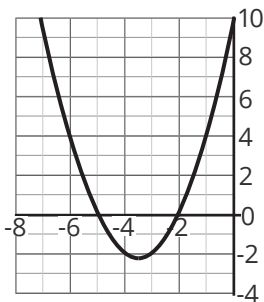
$$1000x = 3165.6565\dots$$

$$10x = 31.656565\dots$$

$$990x = 3134$$

$$x = \frac{3134}{990} = \frac{1567}{495}$$

6. Solve $x^2 + 7x + 10 < 0$



$$(x + 2)(x + 5) < 0$$

$$-5 < x < -2$$

7. Prove algebraically that the sum of two consecutive odd numbers is a multiple of 4.

Let $2n + 1$ and $2n + 3$ be consecutive odd numbers for integer values of n .

$$2n + 1 + 2n + 3 = 4n + 4$$

$4(n + 1)$ is, by definition, a multiple of 4.

8. Make x the subject: $ax + bx = t$

$$x(a + b) = t$$

$$x = \frac{t}{a + b}$$

9. A line segment starts at the point (1, 3) and finishes at (5, 6). Find the equation of the perpendicular bisector to this line segment.

$$m_1 = \frac{6 - 3}{5 - 1} = \frac{3}{4}$$

$$m_2 = -\frac{4}{3}$$

Midpoint is (3, 4.5).

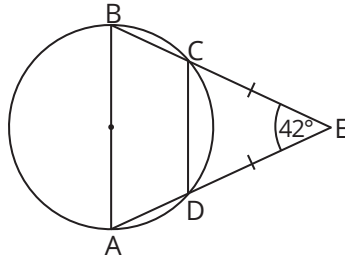
$$y = -\frac{4}{3}x + c$$

$$4.5 = -\frac{4}{3} \times 3 + c$$

$$c = \frac{17}{2}$$

$$y = -\frac{4}{3}x + \frac{17}{2}$$

10. ADE and BCE are straight lines. Calculate the size of angle ABC.

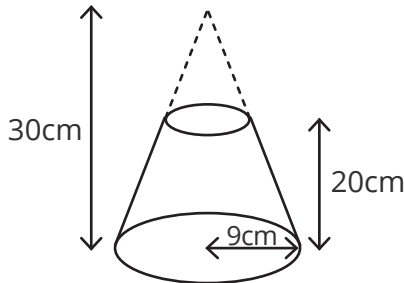


$$\text{Angle CDE} = \frac{180 - 42}{2} = 69^\circ$$

$$\text{Angle CDA} = 180 - 69 = 111^\circ$$

$$\text{Angle ABC} = 180 - 111 = 69^\circ$$

11. The diagram shows the frustum of a cone. The height of the cone is 30cm and the height of the frustum is 20cm. The radius of the base of the cone is 9cm. Given that the mass of the frustum is 1000 grams, work out its density, giving your answer correct to 1 decimal place.



$$\text{Scale factor} = 30 \div 10 = 3$$

$$\text{Radius of 'top' cone} = 9 \div 3 = 3\text{cm}$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume} = \frac{1}{3} \times \pi \times 9^2 \times 30 - \frac{1}{3} \times \pi \times 3^2 \times 10 = 780\pi\text{cm}^3$$

$$\text{Density} = 1000 \div 780\pi = 0.4\text{g/cm}^3$$

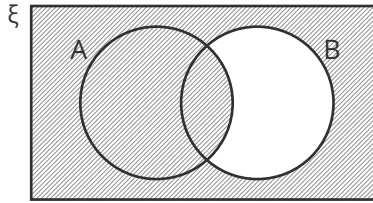
12. $a:b = \frac{1}{2}:\frac{1}{3}$ and $b:c = \frac{2}{3}:4$. Write a as a fraction of c , giving your answer in the form $a = \frac{x}{y}c$, where x and y are integers.

$$a:b = 1:\frac{2}{3}$$

$$a:c = 1:4$$

$$a = \frac{1}{4}c$$

13. On the Venn diagram, shade $A \cup B'$.



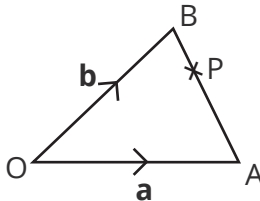
14. Solve $x(3x + 2) = x + 1$, giving your answer correct to 1 decimal place.

$$3x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 3 \times (-1)}}{2 \times 3}$$

$$x = 0.4 \text{ or } -0.8$$

15. $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. Point P lies on AB such that $AP:PB = 3:2$. Write down the vector \vec{OP} .



$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

$$\vec{AP} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a}) = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$$

16. Solve the simultaneous equations, giving your answers in terms of a and b .

$$x + y = a$$

$$x - y = b$$

$$2x = a + b$$

$$x = \frac{1}{2}(a + b)$$

$$\frac{1}{2}(a + b) + y = a$$

$$y = \frac{1}{2}(a - b)$$

17. Given that $27^{2x+5} = 3^y$, express y in terms of x .

$$27^{2x+5} = (3^3)^{2x+5} = 3^{6x+15}$$

$$y = 6x + 15$$

18. Solve the equation $\sin(x) = -0.1$ for $0^\circ \leq x \leq 360^\circ$.

$$x = \sin^{-1}(-0.1) = -5.7^\circ$$

$$x = 185.7^\circ, 354.3^\circ \text{ (correct to 1d.p.)}$$

19. Work out the value of $4.9 \times 10^4 - 1.1 \times 10^2$, giving your answer in standard form.

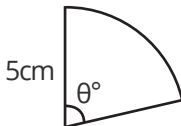
$$48\,890 = 4.889 \times 10^4$$

20. The table shows information about the ages of some employees in a company. Fill in the missing gaps.

| Age (x years) | Frequency | Frequency Density |
|------------------|----------------------|-------------------|
| $16 \leq x < 20$ | 14 | $14 \div 4 = 3.5$ |
| $20 \leq x < 30$ | $2.8 \times 10 = 28$ | 2.8 |
| $30 \leq x < 50$ | 6 | $6 \div 20 = 0.3$ |
| $50 \leq x < 55$ | $1.6 \times 5 = 8$ | 1.6 |
| $55 \leq x < 70$ | 3 | $3 \div 15 = 0.2$ |

1. Simplify $\frac{x^2 - x - 12}{3x^2 - 7x - 20}$
 $\frac{x + 3}{3x + 5}$

2. The diagram shows a sector of a circle of radius 5cm. Given that the perimeter of the sector is 13.05cm, work out its area, giving your answer correct to 1 decimal place.



$$13.05 - 2 \times 5 = 3.05$$

$$\frac{\theta}{360} \times \pi \times 10 = 3.05 \quad \theta = 34.95\dots^\circ$$

$$\text{Area} = \frac{34.95\dots}{360} \times \pi \times 5^2 = 7.6\text{cm}^2 \text{ (correct to 1 decimal place).}$$

3. Bella invests some money in a bank account which offers 5% interest in the first year, then 2.5% interest compounded annually. She leaves her money in the account for 3 years. What is the single percentage increase in the money in her account?

$$1.05 \times 1.025^2 = 1.1031\dots$$

This is a 10.3% increase correct to 1 decimal place.

4. Factorise fully: $4x^2 - 3x - 10$

$$(4x + 5)(x - 2)$$

5. Write $0.\dot{1}\dot{4} \times 0.\dot{0}\dot{3}$ as a fraction in its simplest form.

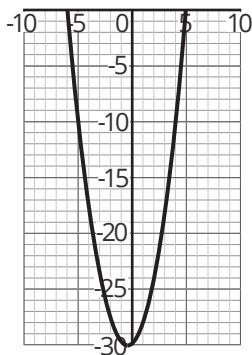
$$0.\dot{1}\dot{4} = \frac{14}{99}$$

$$0.\dot{0}\dot{3} = \frac{3}{99} = \frac{1}{33}$$

$$\frac{14}{99} \times \frac{1}{33} = \frac{14}{3267}$$

$$\frac{14}{99} \times \frac{1}{33} = \frac{14}{3267}$$

6. Solve $x^2 + x - 30 > 0$



$$(x + 6)(x - 5) > 0$$

$$x > 5 \text{ and } x < -6$$

7. Prove algebraically that, when the sum of the squares of two consecutive even numbers is divided by 8, there is a remainder of 4.

Let $2n$ and $2n + 2$ be consecutive even numbers for integer values of n .

$$(2n)^2 + (2n + 2)^2 = 8n^2 + 8n + 4$$

This can be written as $8(n^2 + n) + 4$ which will have a remainder of 4 when divided by 8.

8. Make x the subject: $y + 2x = ax + q$

$$x(2 - a) = q - y$$

$$x = \frac{q - y}{2 - a} \text{ or equivalent}$$

9. A line segment starts at the point $(-1, 4)$ and finishes at $(2, 3)$. Find the equation of the perpendicular bisector to this line segment.

$$m_1 = \frac{3 - 4}{2 - (-1)} = -\frac{1}{3}$$

$$m_2 = 3$$

Midpoint is $(0.5, 3.5)$.

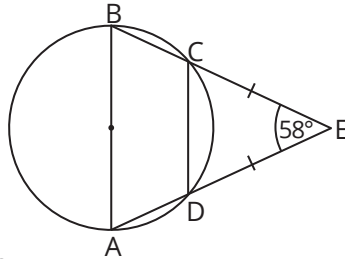
$$y = 3x + c$$

$$3.5 = 3 \times 0.5 + c$$

$$c = 2$$

$$y = 3x + 2$$

10. ADE and BCE are straight lines. Prove that the line segments AB and CD are parallel.



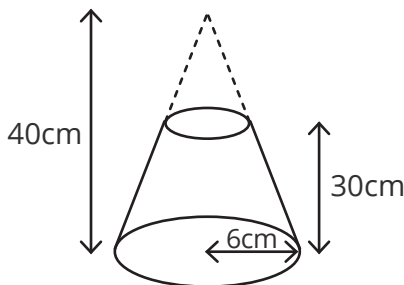
Angle CDE = $\frac{180 - 58}{2} = 61^\circ$ (base angles in an isosceles triangle are equal).

Angle CDA = angle DCB = $180 - 61 = 119^\circ$ (angles on a straight line add to 180°).

Angle ABC = angle $180 - 119 = 61^\circ$ (opposite angles in a cyclic quadrilateral sum to 180°).

Angles ABC and DCB are supplementary (sum to 180°), so AB is parallel to CD.

11. The diagram shows the frustum of a cone. The height of the cone is 40cm and the height of the frustum is 30cm. The radius of the base of the cone is 6cm. Given that the mass of the frustum is 1500 grams, work out its density, giving your answer correct to 2 significant figures.



Scale factor = $40 \div 10 = 4$

Radius of 'top' cone = $6 \div 4 = 1.5\text{cm}$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume = $\frac{1}{3} \times \pi \times 6^2 \times 40 - \frac{1}{3} \times \pi \times 1.5^2 \times 10 = \frac{945}{2}\pi\text{cm}^3$

Density = $1500 \div \frac{945}{2}\pi = 1.01\text{g/cm}^3$

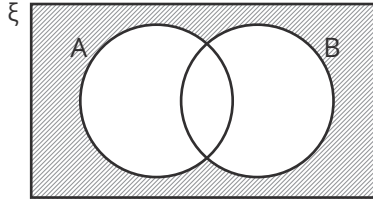
12. $a:b = \frac{1}{2}:\frac{1}{3}$ and $b:c = \frac{2}{3}:\frac{4}{9}$. Write a as a fraction of c , giving your answer in the form $a = \frac{x}{y}c$, where x and y are integers.

$$a:b = 1:\frac{2}{3}$$

$$a:c = 1:\frac{4}{9}$$

$$a = \frac{9}{4}c$$

13. On the Venn diagram, shade $A' \cap B'$.



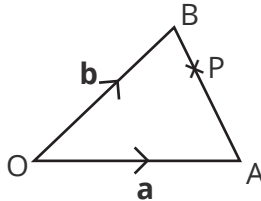
14. Solve $3x(x-2) = 3x+1$, giving your answer correct to 1 decimal place.

$$3x^2 - 9x - 1 = 0$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times -1}}{2 \times 3}$$

$$x = 3.1 \text{ or } -0.1$$

15. $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. Point P lies on AB such that $AP:PB = 3:2$. The point C lies $\frac{3}{4}$ of the way along the line OP. Write down the vector \vec{OC} .



$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

$$\vec{AP} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a}) = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$$

$$\vec{OC} = \frac{3}{4}(\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}) = \frac{3}{10}\mathbf{a} + \frac{9}{20}\mathbf{b}$$

16. Solve the simultaneous equations, giving your answers in terms of x and y .

$$x + y = 5$$

$$x^2 - y^2 = 15$$

$$x = 5 - y$$

$$(5 - y)^2 - y^2 = 15$$

$$25 - 10y = 15$$

$$10y = 10$$

$$y = 1$$

$$x = 4$$

17. Given that $32^{3x} \times 4^{7x} = 2^y$, express y in terms of x .

$$32^{3x} = (2^5)^{3x} = 2^{15x}$$

$$4^{7x} = (2^2)^{7x} = 2^{14x}$$

$$32^{3x} \times 4^{7x} = 2^{15x} \times 2^{14x} = 2^{29x}$$

$$y = 29x$$

18. Solve the equation $\tan(x) = 0.4$ for $0^\circ \leq x \leq 540^\circ$.
Hint: There are three answers.

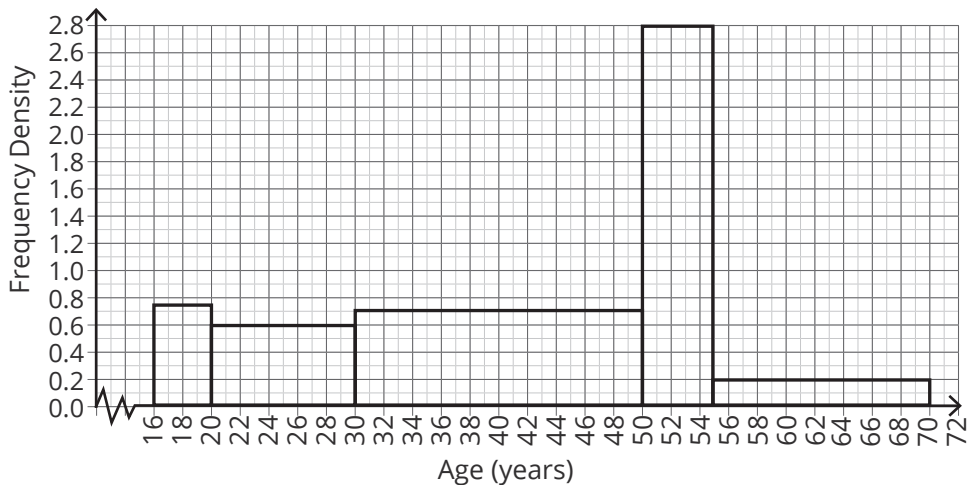
$$x = \tan^{-1}(0.4) = 21.8^\circ$$

$$x = 21.8^\circ, 201.8^\circ, 381.8^\circ \text{ (correct to 1d.p.)}$$

19. Work out the value of $\frac{4.1 \times 10^{-5} + 2.7 \times 10^4}{1.4 \times 10^{-2}}$, giving your answer in standard form correct to 3 significant figures.

$$1\ 928\ 571.432 \approx 1.93 \times 10^6$$

20. The histogram shows information about the ages of some employees in a company. Which class interval contains the median age?



| Age (x years) | Frequency | Frequency Density |
|------------------|----------------------|-------------------|
| $16 \leq x < 20$ | $0.75 \times 4 = 3$ | 0.75 |
| $20 \leq x < 30$ | $0.6 \times 10 = 6$ | 0.6 |
| $30 \leq x < 50$ | $0.7 \times 20 = 14$ | 0.7 |
| $50 \leq x < 55$ | $2.8 \times 5 = 14$ | 2.8 |
| $55 \leq x < 70$ | $0.2 \times 15 = 3$ | 0.2 |

Total frequency = 40

$$\frac{40 + 1}{2} = 20.5$$

The 20.5th person is in the interval $30 \leq x < 50$.

