Answers

Non-Calculator

GCSE Maths

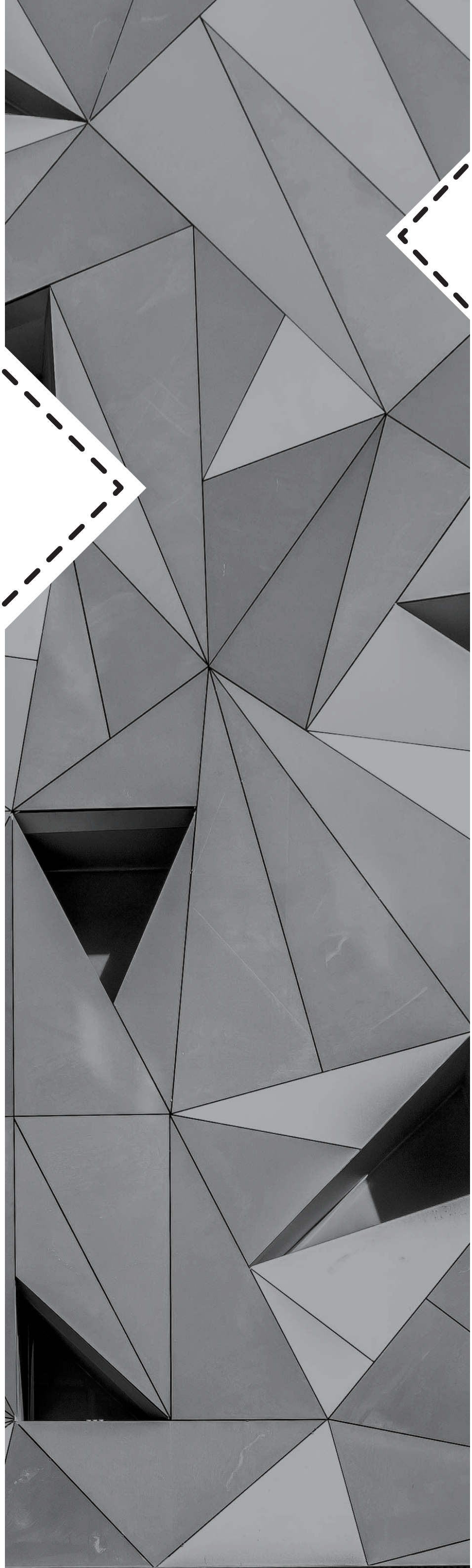
Revision

Higher

Booklet

1

BEYOND MATHS



Non-Calculator

GCSE Maths Revision
Higher Booklet 1 **Answers**

Question Number	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7
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1. A restaurant menu includes 3 starter options and 4 main course options. Assuming a customer chooses one of each, how many different ways are there for them to choose one starter and one main course?

   3 × 4 = 12

2. Expand and simplify: $(x + 5)(x - 4)$

   $\boldsymbol{x^2 + x - 20}$

3. Fill in the gap using the correct inequality symbol: $\sqrt{20}$ __>__ 4

   $\boldsymbol{4 = \sqrt{16}}$ **so** $\boldsymbol{\sqrt{20} > \sqrt{16}}$

4. Given that $f(x) = 7x + 2$, work out $f(-1)$.

   7 × -1 + 2 = -5

5. Evaluate $9^{\frac{1}{2}}$.

   $\boldsymbol{\sqrt{9} = 3}$

6. Write $x^2 + 6x + 10$ in the form $(x + a)^2 + b$, where a and b are integers.

   $\boldsymbol{(x + 3)^2 - 9 + 10 = (x + 3)^2 + 1}$

7. Simplify $\sqrt{75}$.

   $\boldsymbol{\sqrt{25} \times \sqrt{3} = 5\sqrt{3}}$

8. The equation of a circle is $x^2 + y^2 = 49$. What is the radius of this circle?

   $\boldsymbol{\sqrt{49} = 7}$

9. x is directly proportional to y. When $x = 4$, $y = 9$. Write an equation for y in terms of x.

   $\boldsymbol{y = 2.25x}$ **(or equivalent)**

10. 10 fish were captured from a pond, tagged and then released back into the pond. The next day, 20 fish were captured from the same pond. Of these 20 fish, 4 had tags on. Estimate the total population size of the fish.

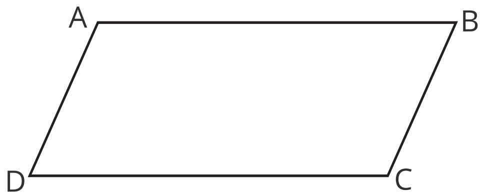
20 ÷ 4 = 5 fish per tag

5 × 10 = 50

11. A function is given by $y = f(x)$. Describe the single transformation that maps this function onto $y = f(x) + 2$.

A translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

12. The diagram shows a parallelogram ABCD. Prove that triangles ABC and ACD are congruent.



There are a number of ways to correctly answer this.

1) AB = CD and AD = BC, since opposite sides in a parallelogram are equal in length. Side AC is a shared side. Therefore, by the SSS condition, the triangles are congruent.

2) AB = CD and AD = BC, since opposite sides in a parallelogram are equal in length. Angle ABC = angle ADC, since opposite angles in a parallelogram are equal. Therefore, by the SAS condition, the triangles are congruent.

3) Side AC is a shared side. Angle BAC = angle ACD and angle ACB = angle CAD, since alternate angles are equal. Therefore, by the ASA condition, the triangles are congruent.

13. Use the iterative formula $x_{n+1} = 3x_n + 2$ with $x_1 = 0.5$ to find the value of x_2.

    $\mathbf{x_2 = 3.5}$

14. The sides of a rectangle are enlarged by a scale factor of 2. Write down the scale factor that the area is enlarged by.

    $\mathbf{2^2 = 4}$

15. The first five terms of a quadratic sequence are 3, 9, 19, 33, 51. Find the n^{th} term of this sequence.

    $\mathbf{2n^2 + 1}$

16. Write down the value of sin(0).

    0

17. A bag contains red and yellow counters only. There are 8 red counters and 4 yellow counters. Ben chooses a counter at random and records the colour. He then replaces it and chooses another. Work out the probability that Ben chooses two red counters.

    $\mathbf{\frac{8}{12} \times \frac{8}{12} = \frac{64}{144} = \frac{4}{9}}$

18. Work out the lower quartile of this list of numbers: 1, 2, 2, 3, 4, 5, 6

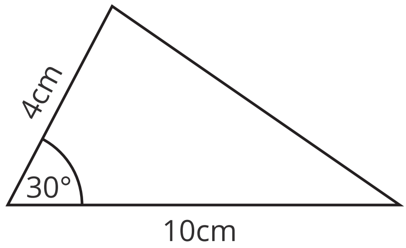
    2

19. A line has equation $y = \frac{1}{2}x + 4$

    Write down the gradient of a line that runs perpendicular to this.

    -2

20. The diagram shows a non-right-angled triangle. Work out its area.



$$\frac{1}{2} \times 10 \times 4 \times \sin(30°) = 10\text{cm}^2$$

1. A restaurant menu includes 2 starter options, 5 main course options and 3 dessert options. Assuming a customer chooses one of each, how many different ways are there for them to choose one starter, one main course and one dessert?

   $2 \times 5 \times 3 = 30$

2. Expand and simplify: $(2x + 3)(x - 1)$

   $2x^2 + x - 3$

3. Fill in the gap using the correct inequality symbol: $\sqrt{31}$ <u><</u> 6

   $6 = \sqrt{36}$ so $\sqrt{31} < \sqrt{36}$

4. Given that $f(x) = 3x^2 - 1$, work out $f(-2)$.

   $3 \times (-2)^2 - 1 = 11$

5. Evaluate $64^{\frac{1}{3}}$.

   $\sqrt[3]{64} = 4$

6. Write $x^2 + 10x + 3$ in the form $(x + a)^2 + b$, where a and b are integers.

   $(x + 5)^2 - 25 + 3 = (x + 5)^2 - 22$

7. Simplify $\sqrt{200}$.

   $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

8. The equation of a circle is $x^2 + y^2 = 9$. Write down the coordinates of the centre of this circle.

   (0, 0)

9. x is directly proportional to y. When $x = 5$, $y = 2$. Write an equation for y in terms of x.

$y = 0.4x$ (or equivalent)

10. 40 fish were captured, tagged and then released. The next day, 100 fish were captured. Of these, 20 had tags. Estimate the total population size of the fish.

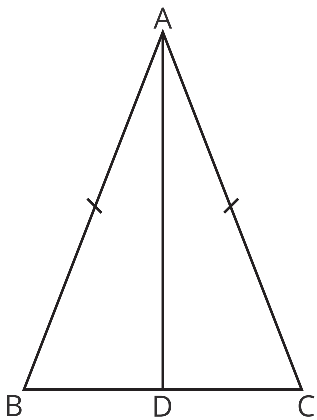
100 ÷ 20 = 5 fish per tag

5 × 40 = 200

11. A function is given by $y = f(x)$. Describe the single transformation that maps this function onto $y = f(x - 1)$.

A translation by the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

12. The diagram shows an isosceles triangle ABC. AD is perpendicular to BD. Prove that triangles ABD and ACD are congruent.



AB = AC and AD is a shared line.

Since AD is perpendicular to BC, angles ADB and ADC are 90˚.

By the RHS condition, the triangles are congruent.

13. Use the iterative formula $x_{n+1} = \sqrt{x_n^2 + 2x_n}$ with $x_1 = 5$ to find the value of x_2. Give your answer correct to 3 significant figures.

(Calculator allowed)

x_2 = 5.92

14. The sides of a cuboid are enlarged by a scale factor of 2. Write down the scale factor that the volume is enlarged by.

$2^3 = 8$

15. The first five terms of a quadratic sequence are 1, 10, 25, 46, 73. Find the n^{th} term of this sequence.

$3n^2 - 2$

16. Write down the value of cos(30°).

$$\frac{\sqrt{3}}{2}$$

17. A bag contains red and yellow counters only. There are 5 red counters and 6 yellow counters. Ben chooses a counter at random and records the colour. He then replaces it and chooses another. Work out the probability that Ben chooses two counters of different colour.

$$\frac{5}{11} \times \frac{6}{11} + \frac{6}{11} \times \frac{5}{11} = \frac{60}{121}$$

18. Work out the upper quartile of this list of numbers:

1, 2, 2, 3, 4, 5, 6, 9, 10, 11, 15

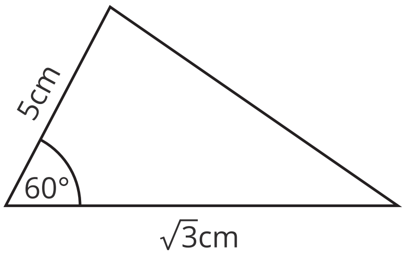
10

19. A line has equation $y = 3x + 7$

Write down the gradient of a line that runs perpendicular to this.

$-\frac{1}{3}$

20. The diagram shows a non-right-angled triangle. Work out its area.



$\frac{1}{2} \times \sqrt{3} \times 5 \times \sin(60°) = \frac{15}{4}\text{cm}^2$

1. Harry has a combination lock. The lock uses 4 digits. Each digit takes a number between 0 and 9 inclusive. How many different ways can this lock be set?

   $10 \times 10 \times 10 \times 10 = 10\,000$

2. Expand and simplify: $(3x + 5)^2$

   $9x^2 + 30x + 25$

3. Estimate the value of $\sqrt{40}$

   $36 < 40 < 49$

   An answer in the interval [6.1, 6.5] would be sufficient.

4. Given that $f(x) = 5x - 4$, work out $f^{-1}(x)$.

   $f^{-1}(x) = \dfrac{x + 4}{5}$

5. Evaluate $8^{\frac{2}{3}}$.

   $\sqrt[3]{8} = 2$

   $2^2 = 4$

6. By writing $x^2 + 6x + 3$ in the form $(x + a)^2 + b$, where a and b are integers, solve the equation $x^2 + 6x + 3 = 0$. Give your answers in surd form.

   $(x + 3)^2 - 9 + 3 = (x + 3)^2 - 6$

   $(x + 3)^2 - 6 = 0$

   $(x + 3)^2 = 6$

   $x + 3 = \pm\sqrt{6}$

   $x = -3 \pm \sqrt{6}$

7. Write $\sqrt{48}$ in the form $a\sqrt{b}$ where a and b are integers.

$\sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

8. The equation of a circle is $x^2 + y^2 = 9$. Does the point (1, 2) lie on the circumference of this circle? Give a reason for your answer.

No. **$1^2 + 2^2 = 5 \neq 9$**

9. x is directly proportional to y. When $x = 5$, $y = 3$. Work out the value of y when $x = 9$.

$y = 0.6x$ (or equivalent)

$y = 0.6 \times 9 = 5.4$

10. 40 fish were captured, tagged and then released. The next day, 100 fish were captured. Of these, 20 had tags. Callum uses this information to estimate the total population of fish. Write down two assumptions he must make.

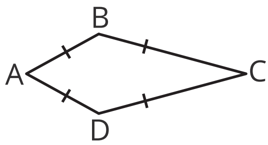
1) The population remains unchanged (i.e. no fish are added or removed),

2) The sample is random.

11. A function is given by $y = f(x)$. Describe the single transformation that maps this function onto $y = -f(x)$.

A reflection in the x-axis or the line $y = 0$.

12. The diagram shows a kite ABCD. Prove that triangles ABC and ADC are congruent.



There are a number of ways to correctly answer this.

1) AB = AD and DC = BC, since adjacent sides in a kite are equal in length. Side AC is a shared side. Therefore, by the SSS condition, the triangles are congruent.

2) AB = AD and DC = BC, since adjacent sides in a kite are equal in length. Angle ABC = angle ADC, since opposite angles in a kite are equal. Therefore, by the SAS condition, the triangles are congruent.

13. Use the iterative formula $x_{n+1} = 7 - \frac{3}{x_n}$ with $x_0 = 1$ to find the value of x_1, x_2 and x_3.

(Calculator allowed)

$x_1 = 4$, $x_2 = 6.25$ **and** $x_3 = 6.52$

14. The sides of a cuboid are enlarged by a scale factor of 4. Write down the scale factor that the surface area is enlarged by.

$4^2 = 16$

15. The first five terms of a quadratic sequence are 5, 14, 27, 44, 65. Find the n^{th} term of this sequence.

$2n^2 + 3n$

16. Write down the value of tan(45°).

1

17. A bag contains red and yellow counters only. There are 5 red counters and 6 yellow counters. Ben chooses a counter at random and records the colour. He then replaces it and chooses another. Work out the probability that Ben chooses at least one red counter.

$\mathbf{\frac{5}{11} \times \frac{6}{11} + \frac{6}{11} \times \frac{5}{11} + \frac{5}{11} \times \frac{5}{11} = \frac{85}{121}}$ **or** $\mathbf{1 - \frac{6}{11} \times \frac{6}{11} = \frac{85}{121}}$

18. Work out the interquartile range of this list of numbers:

3, 7, 8, 9, 9, 11, 13

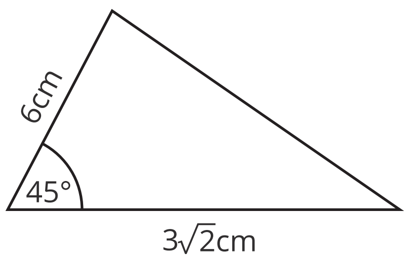
11 – 7 = 4

19. A line has equation $2y = x + 5$

Write down the gradient of a line that runs perpendicular to this.

-2

20. The diagram shows a non-right-angled triangle. Work out its area.



$\mathbf{\frac{1}{2} \times 3\sqrt{2} \times 6 \times \sin(45°) = 9cm^2}$

1. Harry has a combination lock. The lock uses 4 digits. Each digit takes a number between 0 and 9 inclusive. How many different ways can this lock be set, given that Harry wants the code to be even?

   $10 \times 10 \times 10 \times 5 = 5000$

2. Expand and simplify: $(4x - 1)^2$

   $16x^2 - 8x + 1$

3. Estimate the value of $\sqrt{70}$

   $64 < 70 < 81$

   An answer in the interval [8.1, 8.6] would be sufficient.

4. Given that $f(x) = \frac{3x + 1}{2}$, work out $f^{-1}(x)$.

   $f^{-1}(x) = \frac{2x - 1}{3}$

5. Evaluate $9^{\frac{3}{2}}$.

   $\sqrt{9} = 3$

   $3^3 = 27$

6. By writing $x^2 - 4x + 1$ in the form $(x + a)^2 + b$, where a and b are integers, solve the equation $x^2 - 4x + 1 = 0$. Give your answers in surd form.

   $(x - 2)^2 - 4 + 1 = (x - 2)^2 - 3$

   $(x - 2)^2 - 3 = 0$

   $(x - 2)^2 = 3$

   $x + 2 = \pm\sqrt{3}$

   $x = -2 \pm \sqrt{3}$

7. Write $\sqrt{8} + \sqrt{50}$ in the form $a\sqrt{b}$ where a and b are integers.

$\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

$2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$

8. The equation of a circle is $x^2 + y^2 = 13$. Does the point (3, 2) lie on the circumference of this circle? Give a reason for your answer.

Yes. **$3^2 + 2^2 = 13$**

9. x is directly proportional to y. When $x = 4$, $y = 7$. Work out the value of y when $x = 12$.

$y = 1.75x$ (or equivalent)

$y = 1.75 \times 12 = 21$

10. A farmer captures 80 mice on his farm. He tags them and then releases them back onto the farm. The next day, he captures 200 mice. Of these, 50 have tags. Estimate the total population size of the mice.

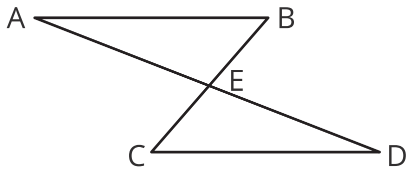
200 ÷ 50 = 4 mice per tag

4 × 80 = 320

11. A function is given by $y = f(x)$. Describe the single transformation that maps this function onto $y = f(-x)$.

A reflection in the y-axis or the line $x = 0$.

12. In the diagram, the lines AB and CD are parallel and AB = CD. Lines BC and AD intersect at point E. Prove that triangles ABE and CDE are congruent.



AB = CD (given).

Angle EAB = angle EDC and angle EBA = angle ECD, since alternate angles are equal.

Therefore, by the ASA condition, the triangles are congruent.

13. Use the iterative formula $x_{n+1} = 7 - \frac{3}{x_n}$ with $x_1 = 1$ to solve the equation $x = 7 - \frac{3}{x}$, giving your answer correct to three decimal places.

(Calculator allowed)

$\mathbf{x = 6.541}$

14. The sides of a cuboid are enlarged by a scale factor of 4. Write down the scale factor that the volume is enlarged by.

$\mathbf{4^3 = 64}$

15. The first five terms of a quadratic sequence are 3, 12, 25, 42, 63. Find the n^{th} term of this sequence.

$\mathbf{2n^2 + 3n - 2}$

16. Write down the value of cos(45°).

$\mathbf{\frac{\sqrt{2}}{2}}$

17. A bag contains red and yellow counters only. There are 5 red counters and 6 yellow counters. Ben chooses a counter at random and records the colour. He then chooses another without replacing the first. Work out the probability that Ben chooses two red counters.

$\mathbf{\frac{5}{11} \times \frac{4}{10} = \frac{20}{110} = \frac{2}{11}}$

18. Work out the interquartile range of this list of numbers:

15, 8, 4, 5, 19, 2, 10

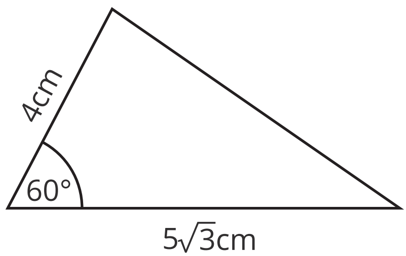
15 – 4 = 11

19. A line has equation $2y - x = 3$

Write down the gradient of a line that runs perpendicular to this.

-2

20. The diagram shows a non-right-angled triangle. Work out its area.



$\mathbf{\frac{1}{2} \times 5\sqrt{3} \times 4 \times \sin(60°) = 15cm^2}$

1. Kelly thinks of a three-digit number greater than or equal to 700. Given that the number is a multiple of 5, calculate the total amount of numbers Kelly could be thinking of.

   3 × 10 × 2 = 60

2. Expand and simplify: $(x + 1)(x + 2)(x + 3)$

   $\mathbf{x^3 + 6x^2 + 11x + 6}$

3. Estimate the value of $\sqrt[3]{52}$

   27 < 52 < 64

   An answer in the interval [3.5, 3.9] would be sufficient.

4. Given that $f(x) = x + 2$ and $g(x) = 3x - 1$, work out $fg(x)$.

   $\mathbf{fg(x) = 3x - 1 + 2 = 3x + 1}$

5. Evaluate $1000^{\frac{2}{3}}$.

   $\mathbf{\sqrt[3]{1000} = 10}$

   $\mathbf{10^2 = 100}$

6. By writing $x^2 + x - 1$ in the form $(x + a)^2 + b$, where a and b are rational numbers, solve the equation $x^2 + x - 1 = 0$. Give your answers in surd form.

   $\mathbf{(x + \frac{1}{2})^2 - \frac{1}{4} - 1 = (x + \frac{1}{2})^2 - \frac{5}{4}}$

   $\mathbf{(x + \frac{1}{2})^2 - \frac{5}{4} = 0}$

   $\mathbf{(x + \frac{1}{2})^2 = \frac{5}{4}}$

   $\mathbf{x + \frac{1}{2} = \pm\sqrt{\frac{5}{4}}}$

   $\mathbf{x = -\frac{1}{2} \pm \sqrt{\frac{5}{4}}}$ **or** $\mathbf{x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}}$

7. Write $\sqrt{45} + 2\sqrt{20}$ in the form $a\sqrt{b}$ where a and b are integers.

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$2\sqrt{20} = 2 \times \sqrt{4} \times \sqrt{5} = 4\sqrt{5}$

$3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$

8. The equation of a circle is $x^2 + y^2 = 13$. Find the equation of the radius of the circle that passes through the point (3, 2).

Gradient = $\frac{2-0}{3-0} = \frac{2}{3}$

The y-intercept is 0 so the equation is $y = \frac{2}{3}x$.

9. x is inversely proportional to y. When $x = 9$, $y = 2$. Work out the value of y when $x = 4$.

$y = \frac{18}{x}$

$y = \frac{18}{4} = 4.5$ (or equivalent)

10. On Monday, Lily captured 320 trout, tagged them and then released them. On Tuesday, Lily captured 500 trout and recorded the number which were tagged. She uses the information to estimate the total amount of trout as 8000. How many trout did Lily record as tagged on Tuesday?

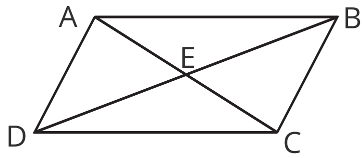
8000 ÷ 320 = 25 trout per tag

500 ÷ 25 = 20 trout had tags

11. A function is given by $y = f(x)$. The graph of $y = f(x)$ passes through the point P with coordinates (1, 2). The function is mapped onto $y = f(x + 5)$. Write down the new coordinates of point P.

(-4, 2)

12. ABCD is a parallelogram. The diagonals of the parallelogram meet at point E. Prove that triangle ABE is congruent to triangle CDE.



There are a number of ways to correctly answer this.

1) AB = CD, since opposite sides in a parallelogram are equal in length.

AE = EC and DE = BE, since diagonals of a parallelogram bisect one another.

By the SSS condition, the triangles are congruent.

2) Angle AEB = angle DEC since vertically opposite angles are equal.

AE = EC and DE = BE, since diagonals of a parallelogram bisect one another.

By the SAS condition, the triangles are congruent.

13. Use the iterative formula $x_{n+1} = \frac{1 - x_n^2}{5}$ with $x_1 = 0.5$ to solve the equation $x^2 + 5x - 1 = 0$, giving your answer correct to three decimal places.

x = 0.193 *(Calculator allowed)*

14. A cylinder has a radius of 2cm and a surface area of 10cm^2.
A similar cylinder has a radius of 4cm. Calculate its surface area.

$10 \times 2^2 = 40\text{cm}^2$

15. The first five terms of a quadratic sequence are -3, -3, 1, 9, 21. Find the n^{th} term of this sequence.

$2n^2 - 6n + 1$

16. Evaluate sin(60°) × cos(30°).

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$$

17. A bag contains red and yellow counters only. There are 5 red counters and 6 yellow counters. Ben chooses two counters at random and records the colours. Work out the probability that Ben chooses two counters of the same colour.

$$\frac{5}{11} \times \frac{4}{10} + \frac{6}{11} \times \frac{5}{10} = \frac{50}{110} = \frac{5}{11}$$

18. Work out the lower quartile of this list of numbers: 3, 4, 1, 2, 9

$$\frac{5+1}{4} = 1.5$$

The 1.5th number is 1.5.

19. Find the equation of the line parallel to $y = 2x + 1$ that passes through (1, 1).

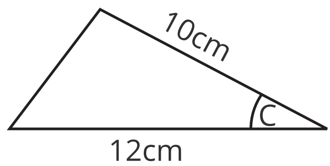
$y = 2x + c$

$1 = 2 \times 1 + c$

$c = -1$

$y = 2x - 1$

20. The diagram shows a non-right-angled triangle. Its area is $30\sqrt{3}$ square units. Find the size of angle C.



$$\frac{1}{2} \times 10 \times 12 \times \sin(C) = 30\sqrt{3}$$

$$\sin(C) = \frac{\sqrt{3}}{2}$$

C = 60°

1. A menu has a choice of 3 starters, 4 main courses and 2 desserts. How many different ways are there to choose a two-course meal (starter and main or main and dessert)?

   $3 \times 4 + 4 \times 2 = 20$

2. Expand and simplify: $(x - 3)(x + 4)(x - 5)$

   $x^3 - 4x^2 - 17x + 60$

3. Estimate the value of $\sqrt[3]{100}$

   $64 < 100 < 125$

   An answer in the interval [4.4, 4.9] would be sufficient.

4. Given that $f(x) = x^2$ and $g(x) = x + 2$, work out $fg(x)$.

   $fg(x) = (x + 2)^2$ or equivalent.

5. Evaluate $64^{-\frac{3}{2}}$.

   $\sqrt{64} = 8$

   $8^{-3} = \frac{1}{512}$

6. By writing $x^2 + 3x - 1$ in the form $(x + a)^2 + b$, where a and b are rational numbers, find the coordinates of the turning point of the curve $y = x^2 + 3x - 1$

   $(x + \frac{3}{2})^2 - \frac{9}{4} - 1 = (x + \frac{3}{2})^2 - \frac{13}{4}$

   The coordinates of the turning point are $(-\frac{3}{2}, -\frac{13}{4})$.

7. Rationalise the denominator of $\frac{10}{\sqrt{5}}$

   $2\sqrt{5}$

8. The equation of a circle is $x^2 + y^2 = 13$. Find the equation of the tangent to the circle that passes through the point (3, 2).

Gradient of radius = $\frac{2-0}{3-0} = \frac{2}{3}$ **Gradient of tangent = $-\frac{2}{3}$**

$y = -\frac{3}{2}x + c$

$2 = -\frac{3}{2} \times 3 + c$

$c = \frac{13}{2}$

$y = -\frac{3}{2}x + \frac{13}{2}$

9. x is inversely proportional to y. When $x = 3$, $y = 4$. Work out the value of x when $y = 10$.

$y = \frac{12}{x}$

$10 = \frac{12}{x}$

x = 1.2 (or equivalent)

10. On Monday, Lily captured 70 trout, tagged them and then released them. On Tuesday, Lily captured 120 trout and recorded the number which were tagged. She uses the information to estimate the total amount of trout as 210. How many trout did Lily record as tagged on Tuesday?

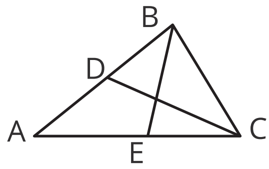
210 ÷ 70 = 3 trout per tag

120 ÷ 3 = 40 trout had tags

11. A function is given by $y = f(x)$. The graph of $y = f(x)$ passes through the point P with coordinates (1, 2). The function is mapped onto $y = -f(x) + 2$. Write down the new coordinates of point P.

(1, 0)

12. In the diagram, AB = AC. D is the midpoint of AB and E is the midpoint of AC. Prove that triangle ABE is congruent to triangle ACD.



AB = AC (given).

Angle BAE = angle CAD since it's a shared angle.

AD = AE since D is the midpoint of AB and E is the midpoint of AC and AB = AC.

By the condition SAS, the triangles are congruent.

13. Show that the equation $x^4 - 4x + 2 = 0$ has a solution in the interval [0, 1].

$\mathbf{0^4 - 4 \times 0 + 2 = 2}$

$\mathbf{1^4 - 4 \times 1 + 2 = -1}$

A change of sign indicates a solution lies in this interval.

14. A cylinder has a radius of 2cm and a volume of 30cm^3. A similar cylinder has a radius of 6cm. Calculate its volume.

$\mathbf{6 \div 2 = 3}$

$\mathbf{30 \times 3^3 = 810\text{cm}^3}$

15. The first five terms of a quadratic sequence are 3, 8, 15, 24, 35. Find the 50th term of this sequence.

The n^{th} term is $x^2 + 2x$.

$\mathbf{50^2 + 2 \times 50 = 2600}$

16. Which of these cannot be the sine of an angle?

$0.5, \frac{\sqrt{3}}{2}, 4, -1$ **4**

17. A biscuit tin contains 4 chocolate and 8 plain biscuits only. Alex chooses two biscuits at random. Work out the probability that at least one of these biscuits is plain.

$$\frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11} + \frac{8}{12} \times \frac{7}{11} = \frac{120}{132} = \frac{10}{11} \text{ or } 1 - \frac{4}{12} \times \frac{3}{11} = \frac{10}{11}$$

18. Work out the upper quartile of this list of numbers: 3, 4, 1, 2, 9

$$3 \times \frac{5 + 1}{4} = 4.5$$

The 4.5th number is 6.5.

19. Find the equation of the line perpendicular to $y = 2x + 1$ that passes through (1, 1).

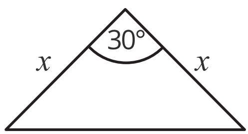
$y = -\frac{1}{2}x + c$

$1 = -\frac{1}{2} \times 1 + c$

$c = \frac{3}{2}$

$y = -\frac{1}{2}x + \frac{3}{2}$

20. The diagram shows a non-right-angled isosceles triangle. Its area is 9 square units. Calculate the value of x.



$\frac{1}{2} \times x \times x \times \sin(30°) = 9$

$x^2 = 36$

$x = 6$ **units**

1. A hotel has five rooms to allocate to five separate guests. How many different ways are there for the hotel to do this?

   $5 \times 4 \times 3 \times 2 \times 1 = 120$

2. Expand and simplify $(x + 3)^3$

   $x^3 + 9x^2 + 27x + 27$

3. Estimate the value of $\sqrt{\frac{10}{24}}$, giving your answer as a fraction.

   $9 < 10 < 16$

   $16 < 24 < 25$

   A sensible answer would be $\frac{3}{5}$

4. Given that $f(x) = x^2 + 3$ and $g(x) = 2x - 1$, work out $fg(5)$.

   $fg(x) = (2x - 1)^2 + 3$

   $fg(5) = 84$

5. Evaluate $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$.

   $\sqrt{\frac{9}{16}} = \frac{3}{4}$

   $\left(\frac{3}{4}\right)^{-3} = \frac{64}{27}$

6a. By writing $x^2 + 4x - 3$ in the form $(x + a)^2 + b$, where a and b are integers, find the coordinates of the turning point of the curve $y = x^2 + 4x - 3$

   $(x + 2)^2 - 4 - 3 = (x + 2)^2 - 7$

   The coordinates of the turning point are (-2, -7).

6b. Solve the equation $x^2 + 4x - 3 = 0$, giving your answers in surd form.

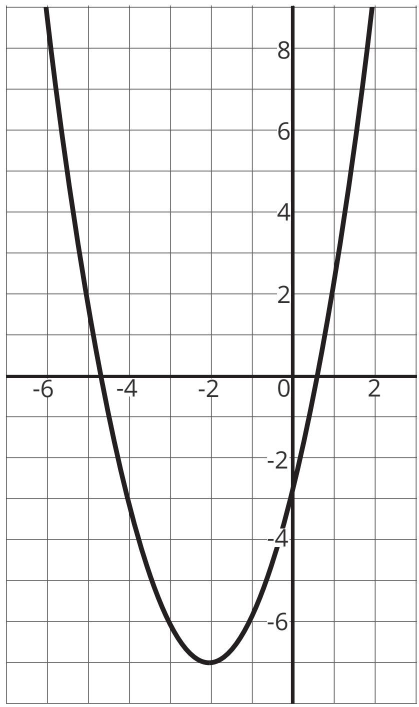
$$(x + 2)^2 - 7 = 0$$

$$(x + 2)^2 = 7$$

$$x + 2 = \pm\sqrt{7}$$

$$x = -2 \pm \sqrt{7}$$

6c. Hence, sketch the graph of $y = x^2 + 4x - 3$



7. Rationalise the denominator of $\frac{3}{1 + \sqrt{2}}$

$$\frac{3}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = \frac{3 - 3\sqrt{2}}{1 - 2} = -3 + 3\sqrt{2}$$

8. The equation of a circle is $x^2 + y^2 = 17$. Find the equation of the tangent to the circle that passes through the point (-1, 4).

Gradient of radius = $\frac{4-0}{-1-0}$ **= -4** **Gradient of tangent =** $\frac{1}{4}$

$y = \frac{1}{4}x + c$

$4 = \frac{1}{4} \times -1 + c$

$c = \frac{17}{4}$

$y = \frac{1}{4}x + \frac{17}{4}$

9. A is directly proportional to the square of b. When A = 5, b = 2. Given that $b > 0$, find the value of b when A = 11.25.

A = 1.25b^2

11.25 = 1.25b^2

b **= 3**

10. On Monday, Lily captured some trout from a pond, tagged them and then released them back into the pond.

On Tuesday, Lily captured 20 less trout than on Monday. She found that 10 of these trout were tagged.

She uses the information to estimate the total amount of trout and finds that this is three times the amount of trout she caught on Monday. How many trout did Lily record as tagged on Tuesday?

number of trout caught on Monday = x

number of trout caught on Monday = x **– 20**

estimated total number of trout in pond = 3x

proportion of tagged trout on Monday = $\frac{x}{3x} = \frac{1}{3}$

proportion of tagged trout on Tuesday = $\frac{10}{x - 20}$

$\frac{1}{3} = \frac{10}{x - 20}$

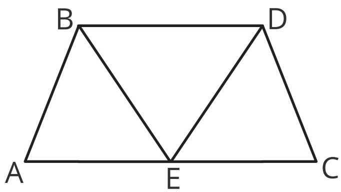
$x - 20 = 30$

Lily recorded 30 tagged trout on Tuesday.

11. A function is given by $y = f(x)$. The graph of $y = f(x)$ passes through the point P with coordinates (1, 2). The function is mapped onto $y = f(x + 4) - 1$. Write down the new coordinates of point P.

(-3, 1)

12. ABCD is a trapezium such that AB = CD. E is the midpoint of AC. Prove that triangle ABE is congruent to triangle DCE.



AB = CD (given).

AE = CE since E is the midpoint of AC.

The trapezium is isosceles so angle BAE = angle DCE.

By the condition SAS, the triangles are congruent.

13. The equation $x^4 - 4x + 2 = 0$ has a solution in the interval [0, 1].

a. Show that the equation can be written as $x = \sqrt[3]{4 - \frac{2}{x}}$

$x^4 = 4x - 2$

$x^3 = 4 - \frac{2}{x}$

$x = \sqrt[3]{4 - \frac{2}{x}}$

b. Use the iterative formula: $x_{n+1} = \sqrt[3]{4 - \frac{2}{x_n}}$ with $x_0 = 1$ to find a solution to the equation $x^4 - 4x + 2 = 0$, giving your answer correct to 2 decimal places.

x_1 = 1.259...

x_2 = 1.341...

x_3 = 1.358...

x_4 = 1.362...

x = 1.36 (to 2d.p.)

14. A cylinder has a surface area of 12cm^2 and a volume of 20cm^3. A similar cylinder has a surface area of 48cm^2. Calculate its volume.

$48 \div 12 = 4 = 2^2$

$20 \times 2^3 = 160$cm^3

15. The first five terms of a quadratic sequence are -1, -13, -33, -61, -97. Find the 50th term of this sequence.

The n^{th} term is $3 - 4x^2$

$3 - 4 \times 50^2 = -9997$

16. Which of these cannot be the cosine of an angle?

0.3, $\frac{\sqrt{2}}{2}$, -0.5, -3

-3

17. A biscuit tin contains 4 chocolate and 8 plain biscuits only. Alex chooses three biscuits at random. Work out the probability that all three biscuits are plain.

$$\mathbf{\frac{8}{12} \times \frac{7}{11} \times \frac{6}{10} = \frac{336}{1320} = \frac{14}{55}}$$

18. Work out the interquartile range of this list of numbers:

3, 5, 1, 2, 9, 2, 9, 0, 1

7 – 1 = 6

19. Find the equation of the line perpendicular to $3y = x - 4$ that passes through (1, 2).

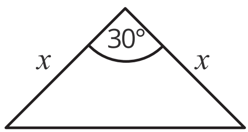
$\mathbf{y = -3x + c}$

$\mathbf{2 = -3 \times 1 + c}$

$\mathbf{c = 5}$

$\mathbf{y = -3x + 5}$

20. The diagram shows a non-right-angled isosceles triangle. Its area is 12 square units. Calculate the value of x, giving your answer as a surd in its simplest form.



$\mathbf{\frac{1}{2} \times x \times x \times \sin(30°) = 12}$

$\mathbf{x^2 = 48}$

$\mathbf{x = 4\sqrt{3}}$ **units**

BEYOND
MATHS